A tree-based approach to price leverage super-senior tranches

Areski Cousin *

November 26, 2009

Abstract

The recent liquidity crisis on the credit derivative market has raised the need for consistent mark-to-model valuation method for some exotic products such as leverage super-senior tranches. Roughly speaking, a Leverage Super-Senior (LSS) tranche is a path-dependent option on the market-value of a traditional super-senior tranche. This option is exercised at the first moment when a particular threshold is hit by a pre-specified trigger proxy. There are three types of proxies commonly used in LSS structures: the pool default losses, the weighted average of CDS spreads and the market-value of the super-senior tranche. We show that the model proposed in Laurent et al. (2007) can be easily adapted to assess the risk of LSS structures and their fair value. In the latter paper, the dynamics of the loss process can be described through a recombining binomial tree and transition probabilities can be calibrated on liquid tranche quotes. In this note, we detail the computation of LSS present values along the nodes of the tree, given that standard option triggers – loss-only trigger, spread trigger or market value trigger – are all stopping times with respect the the loss filtration.

Contents

1 From Markovian contagion models to binomial trees

In the framework of Laurent et al. (2007), the aggregate loss process is assumed to be a continuous-time Markov chain. It is well known that the loss distribution and the

*Post-doctoral researcher, CRIS Project and University of Evry, France, areski.cousin@univ-evry.fr, areski.cousin@gmail.com, http://www.acousin.net. This research was supported by the CRIS (Credit Risk Service) consortium for which we are very grateful.

risk-neutral prices of loss derivative products satisfies the Kolmogorov (resp.) forward and backward equations. When the latter differential equations are discretized, it is straightforward to observe that the price dynamics can be represented along the nodes of a binomial tree. In this section, we explain how to build up this tree.

All random variables introduced so far are defined on the same probability space $(\Omega, \mathscr{G}, \mathbb{Q})$, where $\mathbb{Q}$ is a risk-neutral probability measure. The (fractional) loss at time t is given by $L_t = (1-R)\frac{N_t}{n}$ where R is the recovery rate[1] and n is the number of names in the credit portfolio. In the homogeneous version of the contagion model investigated by Laurent et al. (2007), the number of defaults process N turns to be a continuous-time Markov chain with generator matrix Λ defined by:

$$\Lambda = \begin{pmatrix} -\lambda_0 & \lambda_0 & 0 & & 0 \\ 0 & -\lambda_1 & \lambda_1 & & 0 \\ & & \ddots & \ddots & \\ 0 & & & -\lambda_{n-1} & \lambda_{n-1} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1}$$

The fact that default intensities λ_k, $k = 0, \ldots, n-1$ are assumed to be constant over time is not a limit of our purpose. In the tree approach described below, this assumption can be relaxed without any problem. We denote by $\mathbf{Q}(t,t')$ the transition matrix of N between time t and t', $t < t'$, i.e., $\mathbf{Q}_{k,k'}(t,t') = \mathbb{Q}(N_{t'} = k' \mid N_t = k)$, for all $k,k' \in \{0,\ldots,n\}$.

In what follows, we consider a tranche with attachment point a and detachment point b, $0 \le a \le b \le 1$. We denote by $O(N_t)$ the outstanding nominal on a tranche. It is equal to $b-a$ if $L_t < a$, to $b - L_t$ if $a \le L_t < b$ and to 0 if $L_t \ge b$. For simplicity we assume that the continuously compounded default free interest rate r_t is deterministic and we denote $B(t,t') = \exp\left(-\int_t^{t'} r_s ds\right)$ the time-t discount factor up to time t' $(t \le t')$.

Let us recall that, for a European type payoff the price vector fulfils $V(t,.) = B(t,t')\mathbf{Q}(t,t')V(t',)$ for $0 \le t \le t' \le T$. When the generator matrix Λ does not depend on time, the transition matrix can be expressed as $\mathbf{Q}(t,t') = \exp(\Lambda(t'-t))$.

For practical implementation, we will be given a set of node dates $t_0 = 0, \ldots, t_i, \ldots, t_{n_s} = T$. For simplicity, we will further consider a constant time step $\Delta = t_1 - t_0 = \cdots = t_i - t_{i-1} = \cdots$; this assumption can easily be relaxed. The most simple discrete time approximation one can think of is $\mathbf{Q}(t_i, t_{i+1}) \simeq Id + \Lambda(t_i) \times (t_{i+1} - t_i)$, which leads to $\mathbb{Q}\left(N_{t_{i+1}} = k+1 \,|N_{t_i} = k\right) \simeq \lambda_k \Delta$ and $\mathbb{Q}\left(N_{t_{i+1}} = k \,|N_{t_i} = k\right) \simeq 1 - \lambda_k \Delta$. For large λ_k, the transition probabilities can become negative. Thus, we will rather use the following approximations:

$$\begin{cases} \mathbb{Q}\left(N_{t_{i+1}} = k+1 \,|N_{t_i} = k\right) \simeq 1 - e^{-\lambda_k \Delta}, \\ \mathbb{Q}\left(N_{t_{i+1}} = k \,|N_{t_i} = k\right) \simeq e^{-\lambda_k \Delta}. \end{cases} \tag{2}$$

Given the latter approximations and as illustrated in Figure 1, the dynamics of the number of defaults process can be described through a recombining tree.

[1]The recovery rate is assumed to be the same for all names

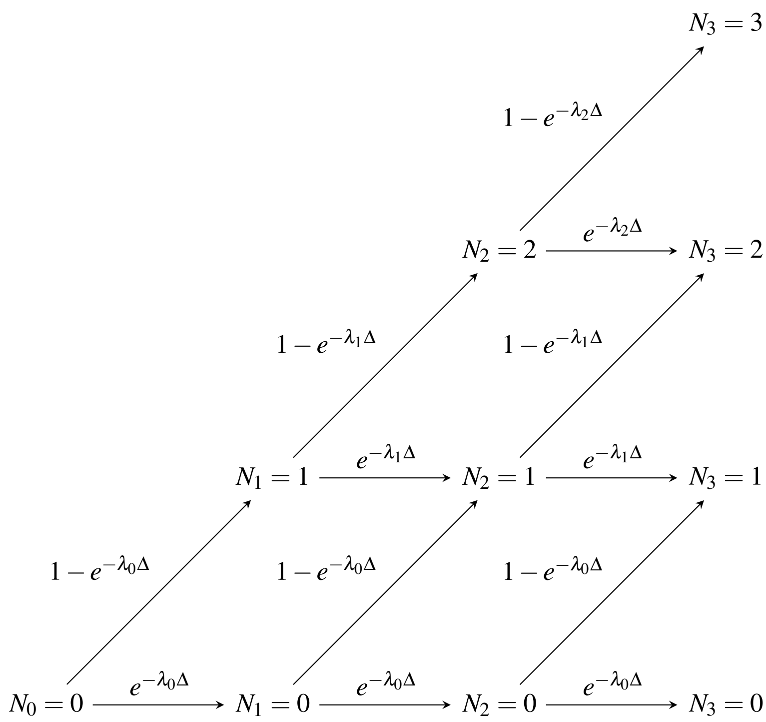

Figure 1: Number of defaults tree

One could clearly think of using continuous Markov chain techniques to compute present values of derivative products at hand, but the tree implementation is quite intuitive from a financial point of view and encompass the pricing of some exotic loss derivatives such as LSS structures. The tree approach corresponds to the implied binomial tree of Derman and Kani (1994). Convergence of the discrete time Markov chain to its continuous limit is a rather standard issue and will not be detailed here.

2 Present values of CDO tranches in the tree nodes

For the sake of notational simplicity, we denote by $B(i) = B(t_i, t_{i+1})$ the value of the discount factor between t_i and t_{i+1}. We also denote by $D(i,k)$ the value at time t_i when $N_{t_i} = k$ of the default payment leg of the CDO tranche[2]. The default payment at time t_{i+1} is equal to $O(N_{t_i}) - O\left(N_{t_{i+1}}\right)$. Thus, $D(i,k)$ is given by the following recurrence equation[3]:

$$D(i,k) = B(i) \cdot \Big(\left(1 - e^{-\lambda_k \Delta}\right) \left[D(i+1,k+1) + O(k) - O(k+1)\right] + e^{-\lambda_k \Delta} D(i+1,k) \Big). \tag{3}$$

[2] We consider the value of the default leg immediately after t_i. Thus, we do not consider a possible default payment at t_i in the calculation of $D(i,k)$.

[3] This relation holds for $i = 0, \ldots, n_s - 1$, $k = 0, \ldots, \min(i, n-1)$ and with $D(n_s, k) = 0$ when $k = 0, \ldots, n$ and $D(i,n) = 0$ when $i = n, \ldots, n_s - 1$.

Let us now deal with a (unitary) premium leg. We denote the regular premium payment dates by $T_1,\dots,T_p$ and for simplicity we assume that: $\left\{T_1,\dots,T_p\right\}\subset\{t_0,\dots,t_{n_s}\}$. Let us consider some date t_{i+1} and set l such that $T_l<t_{i+1}\le T_{l+1}$. Whatever t_{i+1}, there is an accrued premium payment of $\left(O\left(N_{t_i}\right)-O\left(N_{t_{i+1}}\right)\right)\times(t_{i+1}-T_l)$. If $t_{i+1}=T_{l+1}$, i.e., t_{i+1} is a regular premium payment date, there is an extra premium cash-flow at time t_{i+1} of $O\left(N(T_{l+1})\right)\times(T_{l+1}-T_l)$. Thus, if t_{i+1} is a regular premium payment date, the total premium payment is equal to $O\left(N_{t_i}\right)\times(T_{l+1}-T_l)$. Let us denote by $P(i,k)$ the value at time t_i when $N_{t_i}=k$ of the unitary premium leg[4]. If $t_{i+1}\in\left\{T_1,\dots,T_p\right\}$, $P(i,k)$ is provided by:

$$\begin{aligned}P(i,k)=B(i)\cdot\Big(&O(k)\left(T_{l+1}-T_l\right)\\&+\left(1-e^{-\lambda_k\Delta}\right)P(i+1,k+1)+e^{-\lambda_k\Delta}P(i+1,k)\Big).\end{aligned}\tag{4}$$

If $t_{i+1}\notin\left\{T_1,\dots,T_p\right\}$, then[5]:

$$\begin{aligned}P(i,k)=B(i)\cdot\Big(&\left(1-e^{-\lambda_k\Delta}\right)\left[P(i+1,k+1)+\left(O(k)-O(k+1)\right)\left(t_{i+1}-T_l\right)\right]\\&+e^{-\lambda_k\Delta}P(i+1,k)\Big).\end{aligned}\tag{5}$$

The running spread associated with CDO tranche $[a,b]$ is defined by

$$\kappa(i,k)=\frac{D(i,k)}{P(i,k)}.\tag{6}$$

Moreover, the value of the CDO tranche (buy protection case) at time t_i when $N_{t_i}=k$ is given by[6]

$$V(i,k)=D(i,k)-\kappa\cdot P(i,k).\tag{7}$$

where κ is the contractual spread[7].

3 Present values of LSS structures in the tree nodes

Leveraged super-senior structures have an exotic option payout, and are considered to be challenging instruments to value. For a non-formal description of LSS structures mechanism, the reader is referred to Kakodkar et al. (2006), Osako et al. (2005), Kalra et al. (2006) or BIS (2008) for example. As for the pricing of LSS, it has been investigated in different contexts by a number of authors including Sidenius et al. (2005), Hull and White (2006), Brigo et al. (2007), Arnsdorf and Halperin (2007), Walker (2007), Gregory (2008).

[4]As for the default leg, we consider the value of the premium leg immediately after t_i. Thus, we do not take into account a possible premium payment at t_i in the calculation of $P(i,k)$ either.

[5]Relations 4 and 5 hold for $i=0,\dots,n_s-1$, $k=0,\dots,\min(i,n-1)$ and with $P(n_s,k)=0$ when $k=0,\dots,n$ and $P(i,n)=0$ when $i=n,\dots,n_s-1$.

[6]The equity tranche needs to be dealt with slightly differently since its spread is set to $\kappa=500$bp. However, the value of the CDO equity tranche is still given by $D(i,k)-\kappa\cdot P(i,k)$.

[7]The contractual spread is such that the initial market value of the tranche is equal to zero ($V(0,0)=0$). It must be equal to $\kappa=\frac{D(0,0)}{P(0,0)}$.

3.1 Cash-flows of LSS structures

Senior tranches have a low risk of being hit by defaults, and hence pay a relatively low spread to an investor (usually considered to be protection seller). Because an investor in such a tranche will effectively be expected to post a collateral equal to the tranche notional, the return on the invested capital will also be relatively low.

In a leveraged super-senior contract, the investor is liable for a smaller posted collateral, equal to a certain fraction $\alpha < 1$ of the tranche notional. From this perspective, the seller of protection receives the benefit of the cash-flows allocated to the full super senior tranche. The spread leverage is thus equal to $\frac{1}{\alpha}$ compared with the spread paid on the total notional of the tranche.

Because the protection buyer is not protected anymore against default losses that exceed the fraction α of the tranche notional, a specified trigger is introduced. The first time τ that the total basket exceed a pre-specified trigger level, the LSS contract is unwound and settled on a mark-to-market basis, i.e., the investor pays the protection buyer an amount, for an initial tranche notional of unity, equal to:

$$\min\left(\alpha, V_{a,b}(\tau)\right), \tag{8}$$

where $V_{a,b}(\tau)$ is the mark-to-market value of the fully collateralized CDO (un-leveraged tranche) at time τ when the trigger is reached. The unwind trigger is based on pool default losses in the simplest cases, or may be based on average spreads on the reference pool CDS or other proxies such as market value of the un-leveraged senior tranche itself.

In some structures, unwind will be automatic, whereas in others the investor will have the choice to de-leverage via posting more collateral at pre-specified levels. Gregory (2008) argues that when the investor has the choice, it is often optimal for him to unwind the contract and invest in a new LSS structure. In what follows, we assume that investors have the latter behavior at the time when the trigger is hit.

We now describe the computation of LSS present values in the binomial tree depending on what kind of trigger has been chosen.

3.2 Loss-only triggers

Let us consider a LSS structure with leverage $\frac{1}{\alpha}$ referencing a super-senior tranche with attachment point a, detachment point b and maturity T. We denote by $K(t)$ the pre-specified loss-trigger[8]. The loss trigger $K(t)$ is chosen in such a way that the tranche cannot experience losses before the trigger is breached, i.e., $\max_{t \leq T}(K(t)) < a$. We also denote by

$$\tau = \inf\{t \geq 0 \mid L_t \geq K(t)\} \tag{9}$$

the first time at which the loss process exceeds the trigger level[9]. At time τ, the contract is terminated and the protection buyer will receive the value of the un-leverage super

[8]Note that the loss trigger K may be time-dependent. When this is the case, K is usually chosen as a non-decreasing function of time.

[9]Let us remark that τ is a stopping time in the filtration of the loss process.

senior tranche capped at the posted collateral $\alpha(b-a)$. The time-t value of the trigger option given $N_t = k$ is thus equal to:

$$C_{a,b,\alpha}(t,k) = \mathbb{E}\left[\mathbf{1}_{\{\tau<T\}}B(t,\tau)\min\left(V_{a,b}(t,k),\alpha(b-a)\right) \mid N_t = k\right],\ t \geq 0,\ k = 0,\dots,n \tag{10}$$

where $V_{a,b}(t,k)$ is the time-t value of the traditional CDO tranche $[a,b]$ given $N_t = k$. This latter quantity can be computed for every state of the Markov chain N and at every time t using the tree approach described in Section 2.

We now present how to compute $C_{a,b,\alpha}(t,k)$ along the nodes of the binomial tree. Let us define the set I_τ of all nodes (i,k) in tree corresponding to the first times the loss process reaches the trigger level, i.e.,

$$I_\tau = \left\{(i,k) \mid \frac{(1-R)(k-1)}{n} < K(t_i) \leq \frac{(1-R)k}{n}\right\}. \tag{11}$$

In Figure 2, the set I_τ can be represented as the nodes in the tree at the intersection with the trigger curve (in dotted line).

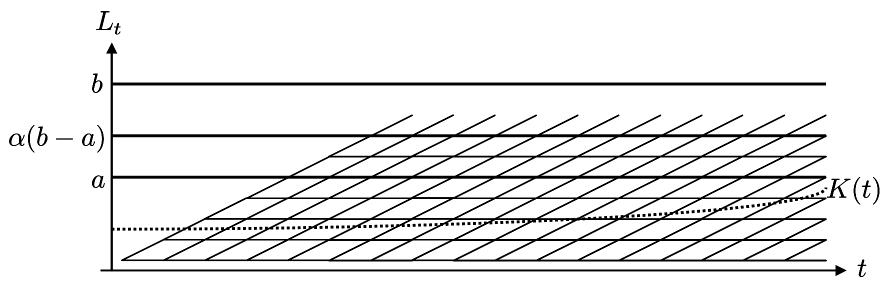

Figure 2: Loss-trigger in the binomial tree

For the sake of notational simplicity, let us denote by $C(i,k) = C_{a,b,\alpha}(t_i,k)$ the value of the trigger option at time t_i, given $N_{t_i} = k$. For all t_i, $i = 0,\dots,n_s$, we denote by $k_\tau(i)$ the number of defaults for which the trigger option is exercised at time t_i[10]. The value of the trigger option at time t_i when $N_{t_i} = k_\tau(i)$ is such that:

$$C(i,k_\tau(i)) = \min\left\{V(i,k_\tau(i)),\alpha(b-a)\right\},\ i = 0,\dots,n_s, \tag{12}$$

where $V(i,k_\tau(i)) = V_{a,b}(t_i,k_\tau(i))$ can be computed using recurrence equation 7. At nodes (i,k) for which $k < k_\tau(i)$, $C(i,k)$ satisfies the following recurrence equation [11]:

$$C(i,k) = B(i)\cdot\left(\left(1-e^{-\lambda_k\Delta}\right)C(i+1,k+1) + e^{-\lambda_k\Delta}C(i+1,k)\right). \tag{13}$$

As for the premium leg, the seller of protection (the investor) receives the benefit of the cash-flows allocated to the full super senior tranche. The only difference compared with the cash-flows generated by a traditional CDO tranche is the fact that the premium payments are obviously stopped after the trigger is being hit because the contract is

[10] $k_\tau(i)$ is such that $(i,k_\tau(i)) \in I_\tau$

[11] This relation holds for $i = 0,\dots,n_s-1$, $k = 0,\dots,\min(i,k_\tau(i)-1)$ and with $C(n_s,k) = 0$ when $k = 0,\dots,n$.

closed after this date. We thus have to adapt recursion formulas 4 and 5 in the following way. Let us denote by J_τ the set of nodes for which the trigger has not been hit yet:

$$J_\tau = \left\{ (i,k) \mid \frac{(1-R)k}{n} < K(t_i) \right\}. \tag{14}$$

Let us denote by $P(i,k)$ the value at time t_i when $N_{t_i} = k$ of the unitary premium leg associated with the LSS. If $t_{i+1} \in \{T_1, \ldots, T_p\}$, $P(i,k)$ is provided by:

$$\begin{aligned} P(i,k) = B(i) \cdot \Big(\left(1 - e^{-\lambda_k \Delta}\right) 1_{(i+1,k+1) \in J_\tau} \left[O(k)\left(T_{l+1} - T_l\right) + P(i+1,k+1) \right] \\ + e^{-\lambda_k \Delta} 1_{(i+1,k) \in J_\tau} \left[O(k)\left(T_{l+1} - T_l\right) + P(i+1,k) \right] \Big). \end{aligned} \tag{15}$$

If $t_{i+1} \notin \{T_1, \ldots, T_p\}$, then[12]:

$$\begin{aligned} P(i,k) = B(i) \cdot \Big(\left(1 - e^{-\lambda_k \Delta}\right) 1_{(i+1,k+1) \in J_\tau} \left[P(i+1,k+1) + \left(O(k) - O(k+1)\right)\left(t_{i+1} - T_l\right) \right] \\ + e^{-\lambda_k \Delta} 1_{(i+1,k) \in J_\tau} P(i+1,k) \Big). \end{aligned} \tag{16}$$

Eventually, market value $V_\alpha(i,k)$ at time t_i given $N_{t_i} = k$ of a buy protection position on this loss-trigger LSS structure, can be expressed as follows:

$$V_\alpha(i,k) = C(i,k) - \kappa \cdot P(i,k), \tag{17}$$

where κ is the contractual spread of the super-senior tranche $[a,b]$, $C(i,k)$ is defined by recurrence equation 13 and $P(i,k)$ is defined by recurrence equations 15 and 16.

3.3 Spread and Market-value triggers

In the case of triggers that are not purely loss-based (spread and market value), we must make a more general analysis to account for the fact that tranche losses may occur before the trigger is hit. This protection leg has the same cash-flows than the default leg on the standard CDO tranche $[a, a+\alpha(b-a)]$ conditional on the trigger event having not previously occurred. As a result the value of the LSS protection is the sum of the following two components, the first corresponding to scenarios before the trigger is hit when the tranche is consumed and the second to the trigger option as before:

$$D_\alpha(t,k) + C_\alpha(t,k), \tag{18}$$

The price $D_\alpha(t,k)$ at time t of the protection on the CDO tranche $[a, a+\alpha(b-a)]$, when $N_t = k$ is given by:

$$D_\alpha(t,k) = \mathbb{E}\left[\int_t^T 1_{s<\tau} B(t,s) dL_s^{(a,a+\alpha(b-a))} \mid N_t = k \right] \tag{19}$$

where $L_t^{(x,y)} = (L_t - x)^+ - (L_t - y)^+$ is the time-t loss on CDO tranche $[x,y]$ with respect to the total loss L_t. If we denote by $D(i,k)$ the value at time t_i when $N_{t_i} = k$ of the LSS

[12] Relations 15 and 16 hold for $i = 0, \ldots, n_s - 1$, $k = 0, \ldots, \min(i, n-1)$ and with $P(n_s, k) = 0$ when $k = 0, \ldots, n$ and $P(i,n) = 0$ when $i = n, \ldots, n_s - 1$.

default payment leg, $D(i,k) = D_\alpha(t_i,k)$ is given by the following recurrence equation[13]:

$$D(i,k) = B(i) \cdot \left(\left(1 - e^{-\lambda_k \Delta}\right) 1_{(i+1,k+1)\in J_\tau} \left[D(i+1,k+1) + O(k) - O(k+1)\right] \right. \\ \left. + e^{-\lambda_k \Delta} 1_{(i+1,k)\in J_\tau} D(i+1,k) \right). \quad (20)$$

where J_τ is the set of nodes for which the trigger has not been hit yet. As this set depends on the type of trigger, it will be defined later on.

Eventually, market value $V_\alpha(i,k)$ at time t_i given $N_{t_i} = k$ of a buy protection position on this loss-trigger LSS structure, can be expressed as follows:

$$V_\alpha(i,k) = D(i,k) + C(i,k) - \kappa \cdot P(i,k), \quad (21)$$

where κ is the contractual spread of the super-senior tranche $[a,b]$, $D(i,k)$ is defined by recurrence equation 20, $C(i,k)$ is defined by recurrence equation 13 and $P(i,k)$ is defined by recurrence equations 15 and 16. Of course, the set of nodes I_τ and J_τ in the latter expressions must be adapted according to the type of trigger under scrutiny.

We now define the sets of nodes I_τ and J_τ used in the computation of V_α in equation 21. These are different according to the type of trigger which is considered: either spread based triggers or market-value based triggers.

3.3.1 Spread triggers

As for a trigger driven by weighted average spreads, we consider that the credit default swap index spread is a good proxy. If κ_t denote the value at time t of the CDS index spread, the unwind date is thus defined by:

$$\tau = \inf\{t \geq 0 \mid \kappa_t \geq K(t, L_t)\} \quad (22)$$

Note that the threshold level may depend both on time and on the current loss state.

The set I_τ of all nodes (i,k) corresponding to the first times the CDS index spread reaches the trigger level is given by:

$$I_\tau = \{(i,k) \mid \kappa(i,k) < K(t_i,k);\ \kappa(i,k+1) \geq K(t_i,k+1)\}, \quad (23)$$

where $\kappa(i,k)$ is the spread of the CDS index at time t_i when $N_{t_i} = k$. The latter quantity can be computed[14] using equation 6 with $a = 0\%$ and $b = 100\%$.

The set J_τ including the nodes for which the trigger has not been hit yet is defined by:

$$J_\tau = \{(i,k) \mid \kappa(i,k) < K(t_i,k)\}. \quad (24)$$

[13] This relation holds for $i = 0,\ldots,n_s-1$, $k = 0,\ldots,\min(i,n-1)$ and with $D(n_s,k) = 0$ when $k = 0,\ldots,n$ and $D(i,n) = 0$ when $i = n,\ldots,n_s-1$.

[14] According to standard market rules, the premium leg of the credit default swap index needs a slight adaptation since the premium payments are based only upon the number of non-defaulted names and do not take into account recovery rates. As a consequence, the outstanding nominal to be used in the recursion equations 4 and 5 providing $P(i,k)$ is such that $O(k) = 1 - \frac{k}{n}$.

3.3.2 Market-value triggers

The last trigger mechanism is based on the mark-to-market of the super-senior tranche. If the value of the tranche falls below a certain threshold, the tranche is unwound at the prevailing market conditions. The unwind date is thus defined by:

$$\tau = \inf\{t \geq 0 \mid V_{a,b}(t) \geq K(t, L_t)\}. \tag{25}$$

where $V_{a,b}(t)$ is the mark-to-market value of the super-senior tranche at time t. Note that the threshold level may depend both on time and on the current loss state.

The set I_τ of all nodes (i,k) corresponding to the first times the tranche market value reaches the trigger level is given by:

$$I_\tau = \{(i,k) \mid V(i,k) < K(t_i,k);\ V(i,k) \geq K(t_i,k+1)\}, \tag{26}$$

where $V(i,k) = V_{a,b}(t_i,k)$ is market value of the super-senior tranche at time t when $N_{t_i} = k$. The latter quantity can be computed using recurrence equation 7.

The set J_τ including the nodes for which the trigger has not been hit yet is defined by:

$$J_\tau = \{(i,k) \mid V(i,k) < K(t_i,k)\}. \tag{27}$$

References

Arnsdorf, M., Halperin, I., 2007. BSLP: Markovian bivariate spread-loss model for portfolio credit derivatives, working paper, JP Morgan.

BIS, 2008. Credit risk transfer: Developments from 2005 to 2007, consultative document, Basel Committee on Banking Supervision.

Brigo, D., Pallavicini, A., Torresetti, R., 2007. Calibration of CDO tranches with the dynamical generalized-Poisson loss model, working paper, Banca IMI.

Derman, E., Kani, I., 1994. Riding on a smile. RISK February, 32–39.

Gregory, J., 2008. A trick of the credit tail. RISK March, 88–92.

Hull, J., White, A., 2006. Dynamic models of portfolio credit risk: A simplified approach, workin paper.

Kakodkar, A., Galiani, S., Jónsson, J., Gallo, A., 2006. Credit Derivatives Handbook 2006 - Vol. 2. Merrill Lynch.

Kalra, R., Yim, G., Hrvatin, R., 2006. ABS leveraged super seniors: Structuring and modeling, Derivative Fitch, Structured Credit.

Laurent, J.-P., Cousin, A., Fermanian, J.-D., 2007. Hedging default risk of CDOs in Markovian contagion models, working paper, ISFA, University of Lyon and BNP Paribas.

Osako, C., Perkins, W., Kissina, I., 2005. Leverage super-senior credit default swaps, FitchRatings, Structured Finance.

Sidenius, J., Piterbarg, V., Andersen, L., 2005. A new framework for dynamic credit portfolio loss modelling, working paper.

Walker, M., 2007. Simultaneous calibration to a range of portfolio credit derivatives with a dynamic discrete-time multi-step Markov loss model, working paper, University of Toronto.