Delta-Hedging Correlation Risk?

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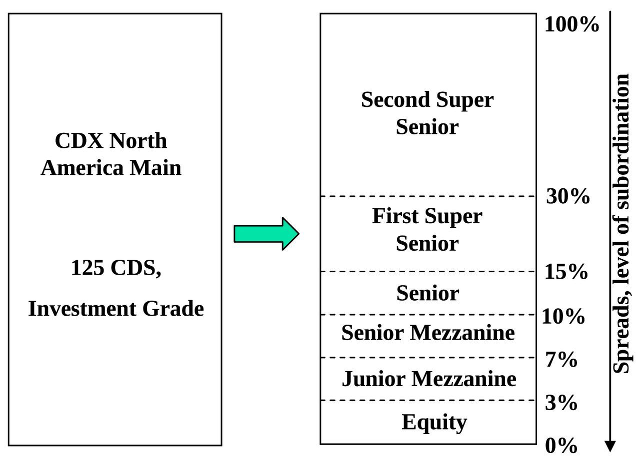
Introduction

- Performance analysis of alternative hedging strategies developed for the correlation market
- CDO tranches on standard CDS index such as CDX North America Investment Grade index



Hedging loss derivatives

- Hedging CDO tranches (buy protection position) on CDS indexes
- Hedging instruments:
  - underlying CDS index
  - savings account

Gaussian copula delta

$$\Delta_t^{\text{Gauss}} = \frac{\mathcal{V}(t, S_t + \varepsilon, \rho_t) - \mathcal{V}(t, S_t, \rho_t)}{\mathcal{V}^I(t, S_t + \varepsilon) - \mathcal{V}^I(t, S_t)}$$

- $\mathcal{V}$: price of the tranche computed in the Gaussian copula model
- $\mathcal{V}^I$: price of the CDX index computed in the Gaussian copula model
- S_t: credit spread of the CDS index at time t
- $\varepsilon = 1$ bp
- ρ_t: implied correlation parameter of the tranche at time t
- Sticky-strike rule: base correlation is kept unchanged when bumping the credit curve

Gauss delta = Sensitivity with respect to the CDS index spread using the industry standard quotation device

Local intensity delta

$$\Delta_t^{\text{lo}} = \frac{V(t, N_t + 1) - V(t, N_t)}{V^I(t, N_t + 1) - V^I(t, N_t)}.$$

- V: price of the tranche computed in the local intensity model
- V^I: price of the CDX index computed in the local intensity model
- N_t: current number of defaults

Local intensity delta = Jump-to-Default delta computed using the local intensity model

Local intensity model

- Laurent, Cousin, Fermanian (2007), Cont and Minca (2008), Cont, Duguest and Kan (2010)
- The number of defaults N_t is modeled as a continuous-time Markov chain (pure birth process) with generator matrix:

$$\Lambda(t) = \begin{pmatrix} -\lambda(t,0) & \lambda(t,0) & 0 & & 0 \\ 0 & -\lambda(t,1) & \lambda(t,1) & & 0 \\ & & \ddots & \ddots & \\ 0 & & & -\lambda(t,n-1) & \lambda(t,n-1) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- $\lambda(t,k)$, $k = 0, \ldots, n-1$: state-dependent default intensities

Model specifications

- **Gauss**: Gaussian copula model with one implied correlation parameter per standard tranche (base correlation approach)
- **Para**: Local intensity model – parametric specification of local itensities

$$\lambda(t,k) = \lambda(k) = (n-k)\sum_{i=0}^{k} b_i$$

Herbertsson (2008)

- **EM**: Local intensity model – local itensities $\lambda(t,k)$ obtained by minimizing a relative entropy distance with respect to a prior distribution

$$\inf_{\mathbb{Q}\in\Lambda} \mathbb{E}^{\mathbb{Q}_0}\left[\frac{d\mathbb{Q}}{d\mathbb{Q}_0}\ln\left(\frac{d\mathbb{Q}}{d\mathbb{Q}_0}\right)\right]$$

Cont and Minca (2008)

Data set and calibration results

- 5-year CDX NA IG Series 5 from 20 September 2005 to 20 March 2006
- 5-year CDX NA IG Series 9 from 20 September 2007 to 20 March 2008
- 5-year CDX NA IG Series 10 from 21 March 2008 to 20 September 2008

Root mean squared calibration errors (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
Index	0.04	5.15	5.14	0.03	4.40	4.81	0.02	6.73	6.77
0%-3%	0.01	2.35	2.36	0.00	1.31	1.32	0.01	1.69	1.68
3%-7%	0.00	0.51	0.69	0.00	0.61	0.86	0.00	1.04	1.03
7%-10%	0.00	0.08	1.32	0.00	0.24	0.91	0.00	0.43	0.39
10%-15%	0.00	0.06	1.77	0.00	0.24	1.15	0.00	0.40	0.36
15%-30%	0.00	0.29	1.97	0.01	1.19	1.74	0.01	1.80	1.68

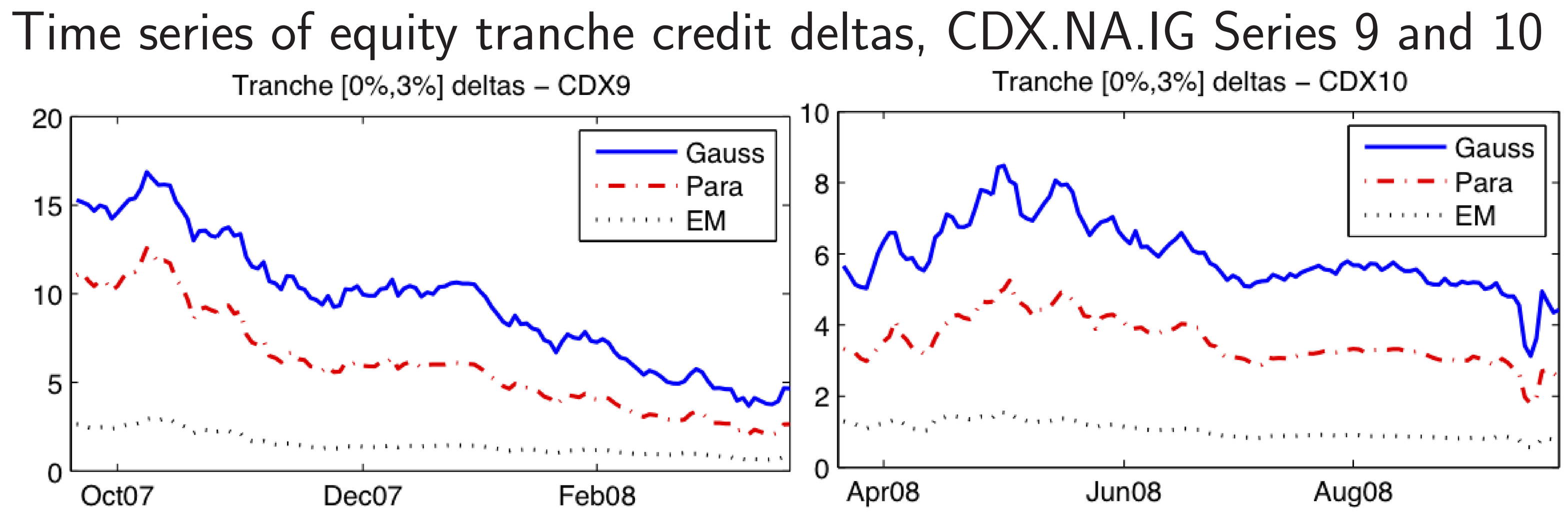
Credit deltas

- Credit deltas on 20 September 2007 (normalized to tranche notional)

Tranche	Δ^{Gauss}	Δ^{lo} Para	Δ^{lo} EM
0%-3%	15.29	11.05	2.64
3%-7%	5.03	4.59	2.70
7%-10%	1.94	2.26	2.29
10%-15%	1.10	1.47	1.99
15%-30%	0.60	1.01	1.74

- The two specifications of the local intensity model (Para and EM) lead to strikingly different deltas ...
- even though calibration is nearly perfect in both approaches
- Significant model risk

- Time series of equity tranche credit deltas, CDX.NA.IG Series 9 and 10



Back-testing hedging experiments

- Two metrics to compare hedging strategies:

$$\text{Relative hedging error} = \left|\frac{\text{Average P\&L increment of the hedged position}}{\text{Average P\&L increment of the unhedged position}}\right|$$

$$\text{Residual volatility} = \frac{\text{P\&L increment volatility of the hedged position}}{\text{P\&L increment volatility of the unhedged position}}$$

- Hedging performance for 5-days rebalancing

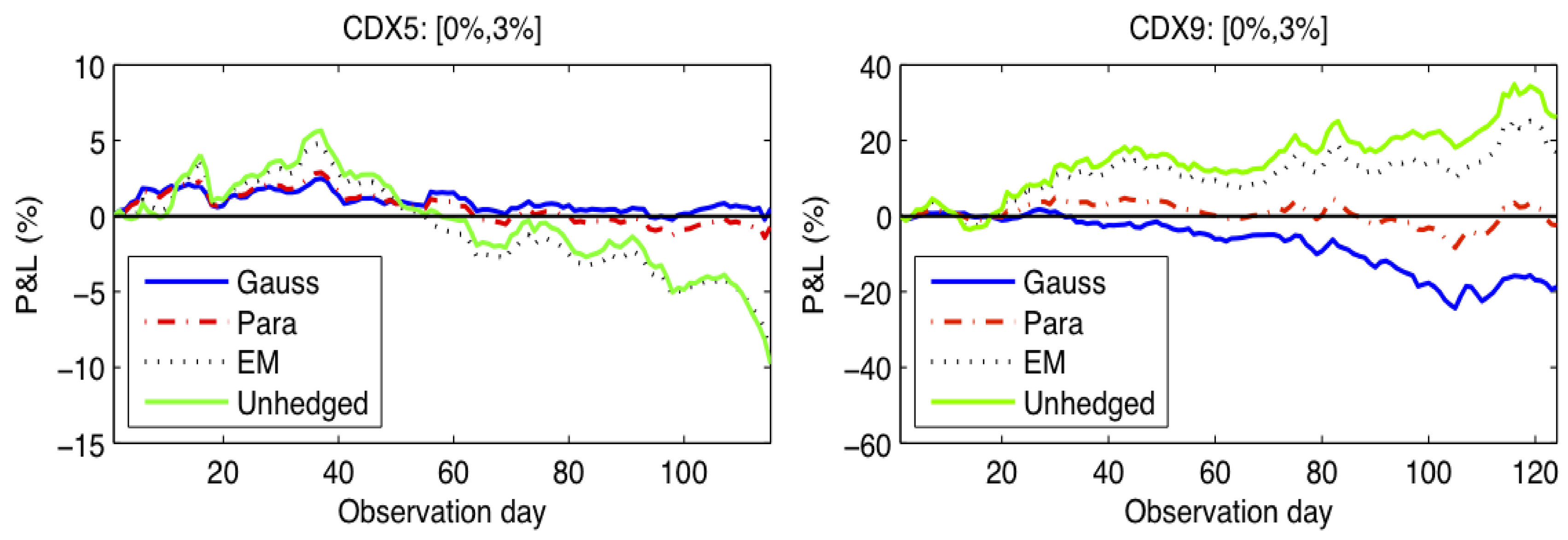
Relative hedging errors (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
0%-3%	6	10	77	59	2	73	24	48	88
3%-7%	16	16	51	2	18	58	48	43	72
7%-10%	19	1	15	11	12	36	50	15	41
10%-15%	22	8	75	13	5	5	141	198	209
15%-30%	21	30	207	1	35	86	127	227	382

Residual volatilities (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
0%-3%	42	46	83	50	56	86	71	72	89
3%-7%	75	75	66	73	65	71	43	40	64
7%-10%	99	118	135	57	56	54	40	38	44
10%-15%	82	110	202	94	98	95	42	44	40
15%-30%	77	108	298	46	69	108	31	33	54

- Path of cumulative P&L of hedged and unhedged positions in equity tranche



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