Delta-Hedging Correlation Risk?

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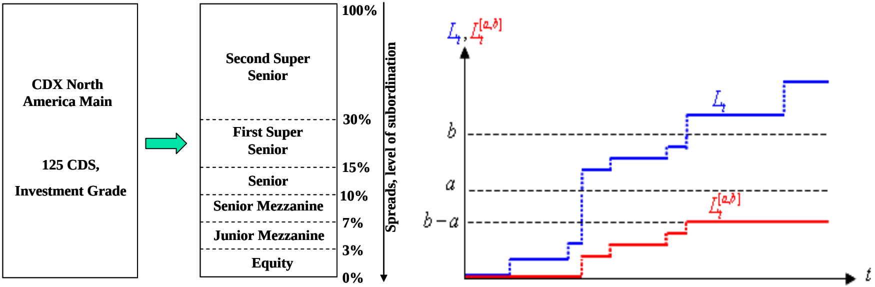
Introduction

Areski Cousin, Stéphane Crépey and Yu Hang Kan (2010)
Delta-Hedging Correlation Risk?

Introduction

- Performance analysis of alternative hedging strategies developed for the correlation market
- CDO tranches on standard Index such as CDX North America Investment Grade index



Introduction

Several risks at hand which may sometimes overlap:

- Default risk of reference entities
  - Cash-flows of synthetic CDO tranches are driven by the evolution of the portfolio loss

$$L_t = \frac{1}{n}\sum_{i=1}^{n}(1 - R_i)\mathbf{1}_{\{\tau_i \leq t\}}$$

- Correlation risk
- Credit spread risk or Market risk
  - Evolution of market prices after inception
- Contagion risk
  - Dynamic combination of credit spread risk and default risk

Hedging loss derivatives

In this study, ...

- We want to hedge of a buy protection position on an index CDO tranche
- Hedging instruments are :
  - CDS Index
  - Savings account

Performance analysis of alternative hedging methods:

- Δ^{Gauss}: delta of the tranche computed within the one-factor Gaussian copula model (industry-standard quotation device)
- Δ^{lo}: delta of the tranche computed within the local intensity model (two specifications of model parameters)

Hedging loss derivatives

Gauss delta:

$$\Delta_t^{\text{Gauss}} = \frac{\mathcal{V}(t, S_t + \varepsilon, \rho_t) - \mathcal{V}(t, S_t, \rho_t)}{\mathcal{V}^I(t, S_t + \varepsilon) - \mathcal{V}^I(t, S_t)}$$

- $\mathcal{V}$: price of the tranche computed in the Gaussian copula model
- $\mathcal{V}^I$: price of the CDX index computed in the Gaussian copula model
- S_t: credit spread of the CDS index at time t
- $\varepsilon = 1$ bp
- ρ_t: implied correlation parameter of the tranche at time t

Gauss delta $=$ Sensitivity with respect to the CDS Index spread using the industry standard quotation device

Hedging loss derivatives

Local intensity delta:

$$\Delta_t^{\text{lo}} = \frac{V(t, N_t + 1) - V(t, N_t)}{V^I(t, N_t + 1) - V^I(t, N_t)}.$$

- V: price of the tranche computed in the local intensity model
- V^I: price of the CDX index computed in the local intensity model
- N_t: current number of defaults

Local intensity delta = Jump-to-Default delta computed using the local intensity model

Local intensity model

- Parallels the Dupire's local volatility approach developed for the equity derivative market
- The number of defaults N_t is modeled as a continuous-time Markov chain (pure birth process) with generator matrix:

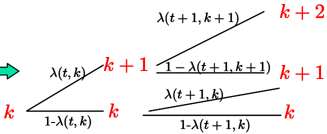
$$\Lambda(t) = \begin{pmatrix} -\lambda(t,0) & \lambda(t,0) & 0 & & 0 \\ 0 & -\lambda(t,1) & \lambda(t,1) & & 0 \\ & & \ddots & \ddots & \\ 0 & & & -\lambda(t,n-1) & \lambda(t,n-1) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

- $\lambda(t,k)$, $k = 0, \dots, n-1$: state-dependent default intensities
- Model involves as many parameters as the number of names

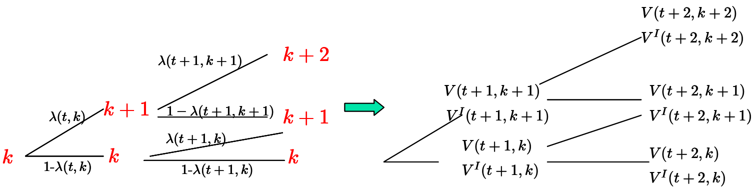
Local intensity model

- Binomial tree: discrete version of the local intensity model

$$\Lambda(t) = \begin{pmatrix} -\lambda(t,0) & \lambda(t,0) & 0 & & 0 \\ 0 & -\lambda(t,1) & \lambda(t,1) & & 0 \\ & & \ddots & \ddots & \\ 0 & & & -\lambda(t,n-1) & \lambda(t,n-1) \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



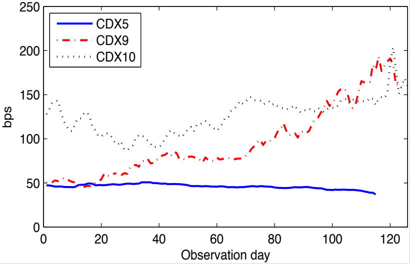
- Given some loss intensities $\lambda(t,k)$, CDO tranches and index prices computed by backward induction:



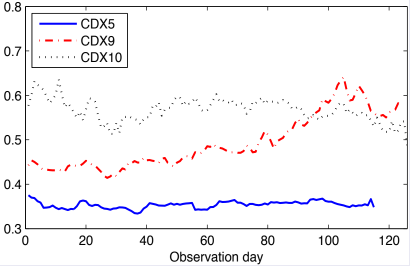
Data set

- 5-year CDX NA IG Series 5 from 20 September 2005 to 20 March 2006
- 5-year CDX NA IG Series 9 from 20 September 2007 to 20 March 2008
- 5-year CDX NA IG Series 10 from 21 March 2008 to 20 September 2008

Index spreads



Base correlation at 3% strike



Model Specifications

- **Gauss**: Gaussian copula model with one implied correlation parameter per standard tranche (base correlation approach)

- **Para**: Local intensity model – parametric specification of local itensities

$$\lambda(t,k) = \lambda(k) = (n-k)\sum_{i=0}^{k} b_i$$

(Herbertsson (2008))

- **EM**: Local intensity model – local itensities $\lambda(t,k)$ obtained by minimizing a relative entropy distance with respect to a prior distribution

$$\inf_{\mathbb{Q}\in\Lambda} \mathbb{E}^{\mathbb{Q}_0}\left[\frac{d\mathbb{Q}}{d\mathbb{Q}_0}\ln\left(\frac{d\mathbb{Q}}{d\mathbb{Q}_0}\right)\right]$$

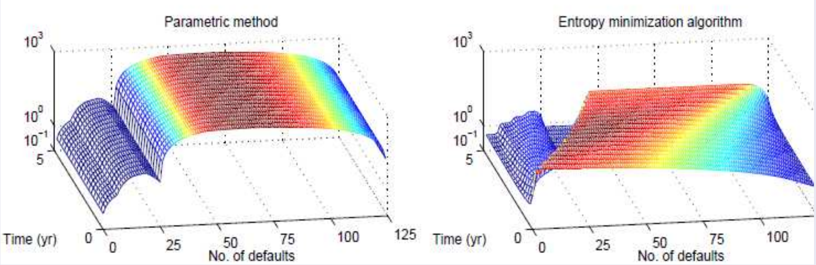
(Cont and Minca (2008))

Empirical results

Root mean squared calibration errors (in percentage):

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
Index	0.04	5.15	5.14	0.03	4.40	4.81	0.02	6.73	6.77
0%-3%	0.01	2.35	2.36	0.00	1.31	1.32	0.01	1.69	1.68
3%-7%	0.00	0.51	0.69	0.00	0.61	0.86	0.00	1.04	1.03
7%-10%	0.00	0.08	1.32	0.00	0.24	0.91	0.00	0.43	0.39
10%-15%	0.00	0.06	1.77	0.00	0.24	1.15	0.00	0.40	0.36
15%-30%	0.00	0.29	1.97	0.01	1.19	1.74	0.01	1.80	1.68

Comparison of typical shapes of local intensities $\lambda(t,k)$, Para (left), EM (right)



Calibration results

Comparison of three alternative hedging methods

- Gauss delta: index Spread sensitivity computed in a one-factor Gaussian copula model

$$\Delta_t^{\text{Gauss}} = \frac{\mathcal{V}(t, S_t + \varepsilon, \rho_t) - \mathcal{V}(t, S_t, \rho_t)}{\mathcal{V}^I(t, S_t + \varepsilon) - \mathcal{V}^I(t, S_t)}$$

  where $\mathcal{V}$ and $\mathcal{V}^I$ are the Gaussian copula pricing function associated with (resp.) the tranche and the CDS index.

- Local intensity delta:

$$\Delta_t^{\text{lo}} = \frac{V(t, N_t + 1) - V(t, N_t)}{V^I(t, N_t + 1) - V^I(t, N_t)}.$$

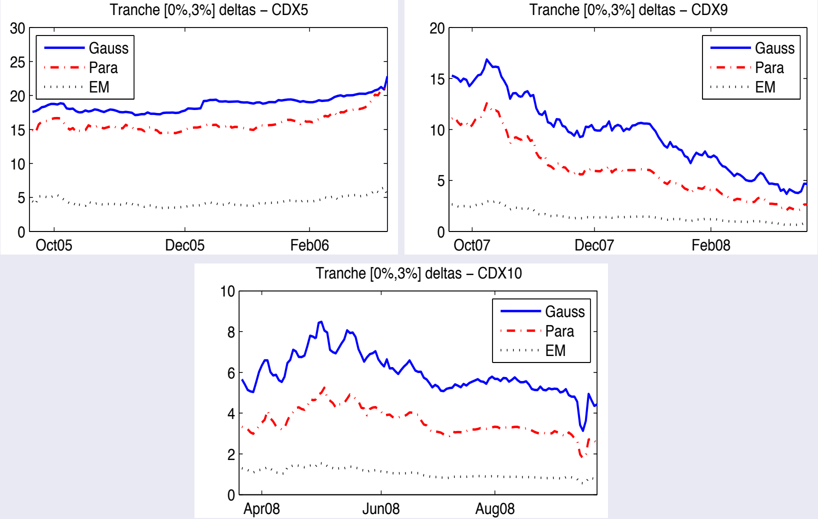
  with both Parametric (Para) and Entropy Minimisation (EM) calibration methods

Credit deltas on 20 September 2007 (normalized to tranche notional)

Tranche	Gauss	Para	EM
0%-3%	15.29	11.05	2.64
3%-7%	5.03	4.59	2.70
7%-10%	1.94	2.26	2.29
10%-15%	1.10	1.47	1.99
15%-30%	0.60	1.01	1.74

Empirical results

Time series of equity tranche credit deltas, CDX.NA.IG series 5, 9 and 10



Hedging performance

Back-testing hedging experiments on series 5, 9 and 10

- Hedging portfolio rebalanced everyday (dt=1)
- P&L (Profit-and-Loss) increment of hedged position:

$$\delta P\&L(t) = \delta V_m(t) - \Delta_t \cdot \delta V_m^I(t)$$

  - $\delta V_m(t) = V_m(t+dt) - V_m(t)$: realized increment of tranche price
  - $\delta V_m^I(t) = V_m^I(t+dt) - V_m^I(t)$: realized increment of index price
  - Δ_t: One of the previous hedging ratios computed at time t

- P&L increments evaluated in the same frequency as rebalancing

Hedging performance

Two metrics to compare the hedging strategies:

$$\textbf{Relative hedging error} = \left|\frac{\text{Average P\&L increment of the hedged position}}{\text{Average P\&L increment of the unhedged position}}\right| = \left|\frac{\text{Average of } \delta P\&L(t)}{\text{Average of } \delta V_m(t)}\right|$$

$$\textbf{Residual volatility} = \frac{\text{P\&L increment volatility of the hedged position}}{\text{P\&L increment volatility of the unhedged position}} = \frac{\text{Volatility of } \delta P\&L(t)}{\text{Volatility of } \delta V_m(t)}$$

Hedging performance for 1-day rebalancing

Relative hedging errors (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Li	Para	EM	Li	Para	EM	Li	Para	EM
0%-3%	4	5	73	80	10	72	33	55	90
3%-7%	1	3	35	0.4	19	59	48	49	75
7%-10%	10	10	43	15	13	37	49	25	44
10%-15%	7	27	131	27	18	14	139	181	208
15%-30%	0.54	61	324	3	32	89	172	269	396

Residual volatilities (in percentage)

	CDX5			CDX9			CDX10		
Tranche	Gauss	Para	EM	Gauss	Para	EM	Gauss	Para	EM
0%-3%	45	47	79	59	59	87	105	91	93
3%-7%	70	72	68	58	47	64	85	74	78
7%-10%	90	101	120	53	50	46	83	79	70
10%-15%	90	107	188	61	63	60	91	93	86
15%-30%	93	110	256	37	49	77	84	99	127

Conclusion

- All model specifications perfectly fit CDO tranche quotes
- However, for the local intensity model, the two introduced specifications give strikingly different deltas and dramatically different hedging performances
- Hedging based on local intensity model with Entropy Minimisation calibration gives poor performance
- Before the crisis (CDX5), Gauss delta outperforms local intensity deltas
- During the crisis (CDX9 & CDX10), no clear evidence to discriminate between Gauss delta and Para local intensity delta